Question Q1.18

If $\vec{\boldsymbol{C}}$ is the vector sum of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$, $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$, what must be true about the directions and magnitudes of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ if $C = A + B$? What must be true about the directions and magnitudes of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ if $C = 0$.

Solution

The vector $\mathbf{C}$ is defined by

$$\mathbf{C} = \mathbf{A} + \mathbf{B}.$$

Take the magnitude of both sides.

$$|\mathbf{C}| = |\mathbf{A} + \mathbf{B}|$$

Assuming that $C = A + B$,

$$|\mathbf{C}| = |\mathbf{A} + \mathbf{B}| = |\mathbf{A}| + |\mathbf{B}|.$$

According to the triangle inequality, $|\mathbf{A} + \mathbf{B}| \leq |\mathbf{A}| + |\mathbf{B}|$. Equality holds only if the vectors have the same direction and sense. Therefore, what's true is that $|\mathbf{C}| = |\mathbf{A}| + |\mathbf{B}|$ and that $\mathbf{A}$ and $\mathbf{B}$ point in the same direction.

If $C = 0$, then

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = \mathbf{0} \quad \rightarrow \quad \mathbf{B} = -\mathbf{A}.$$

This means $\mathbf{A}$ and $\mathbf{B}$ have the same magnitude but are antiparallel (have opposite sense).